Meaning making through comparison of multiplication algorithms: an example of task design in the cultural transposition of lesson study

Alessandro Ramploud, Silvia Funghi and Anna Baccaglini-Frank

Università di Pisa, Italy;

alessandro.ramploud@dm.unipi.it, silvia.funghi@dm.unipi.it, anna.baccaglinifrank@unipi.it

First and second choice theme: number flexibility, number sense

Keywords: Lattice multiplication, lesson study, opaque/transparent algorithm, rectangle/grid multiplication.

Introduction and research questions

This study was carried out within the broader PerContare project (see www.percontare.it) that has the objective of designing, testing, and disseminating inclusive educational practices in primary school mathematics, allowing all children to successfully construct mathematical meanings and prevent and overcome persistent learning difficulties (Baccaglini-Frank, 2015). One of the lines of research that we are exploring involves exposing children to different algorithms, to then create curiosity towards why they "work", leading to their comparison and learning through difference (Bateson, 1979). This specific study is located within this line of research and it involves the teaching and learning of multiplication of natural numbers with two algorithms in third grade (age 8-9).

In the 12 experimental classes on the project, we presented two different multiplication algorithms: multiplication through rectangle diagrams (Mulligan et al., 2018), sometimes referred to as grid multiplication, and lattice multiplication (Siu, 2015). The major difference between the two algorithms that we were interested in can be expressed through the construct of *transparency* with respect to meanings of multiplication between natural numbers. We wish to extend to algorithms the definition of transparent and opaque representations of numbers introduced by Zazkis & Gadowsky (2001): “A transparent representation has no more and no less meaning than the represented idea(s) or structure(s). An opaque representation emphasizes some aspects of the ideas and structures and de-emphasizes others” (Zazkis & Gadowsky, 2001, p. 45). We can say that rectangle diagrams are transparent with respect to how the factors can be decomposed (especially if the decomposition is in base ten, e.g., 51 = 50 + 1), multiplied, then recomposed. On the other hand, lattice multiplication is opaque to such decomposition, allowing the solver to simply multiply single digit numbers and then add them up in columns to generate the digits of the final product.

Our research hypothesis is that children can make sense of the algorithms, understand *why* they work, and gain deep understanding of multiplication of natural numbers, by becoming fluent with both of them and then comparing them and discovering what is behind the opacity of an algorithm like lattice multiplication. Our main research interest is to test this hypothesis. In the study, we report on our attempt at promoting a mathematical discussion overcoming the opacity of lattice multiplication. Specifically, we ask: is it possible in 3rd grade (and if so, how) for children to recognize that in the lattice multiplication of 12 times 14 the "1 × 1" actually represents the operation "10 × 10"?

Methodology

In 2nd grade the children in our experimental classes had discovered multiplication through rectangle diagrams. At the beginning of 3rd grade, for about 3 weeks, the children continued to use rectangle diagrams for multiplication. During these 3 weeks they were also exposed to lattice multiplication, and became familiar with this algorithm, as well, as a procedure. Children were asked which procedure they preferred, and they acknowledged that both procedures lead to the same product, if they were carried out correctly. No classroom tasks were explicitly aimed at meaning making of lattice multiplication until the lesson that was designed to explore our research question.

This lesson was designed by the authors of this paper, together with a group of teachers, following an adaptation of the Lesson Study methodology to Italian classrooms (Bartolini Bussi, Bertolini, Ramploud & Sun, 2017; Mellone, Ramploud, Martignone, & Di Paola, 2019), and it was carried out by the first author. Our adaptation of the Lesson Study methodology refers to the Japanese model, according to which the researchers are quite pro-active (Fujii, 2015); however, as acknowledged by the paradigm of Cultural Transposition (Mellone et al., 2019), a group of 3 teachers also actively engaged in co-designing with 4 researchers the lesson on the comparison of the algorithms.

Outcomes

The lesson was carried out in 9 of the 12 experimental classrooms, but here we limit the discussion to two main findings: 1) an episode from the first lesson design meeting, 2) a key expression regarding how children used rectangle diagrams to shed light onto lattice multiplication.

The phase of lesson design lasted over 2 hours; one of the teachers proposed to ask the children to carry out an activity they were used to: write a number in different ways (e.g., 100 or "one hundred" (1h)) and relate these to the decomposition in powers of 10 in the rectangle diagram, maintaining the cardinality of the numbers involved (e.g., a 10 by 10 rectangle would be made of 100 squares). The group agreed that this could help children relate the "1h" in the lattice multiplication to the 100 in the rectangle diagram. Then the group hypothesized possible students' answers and ways in which the teacher could then respond and promote the discussion. We see this as a cultural transposition of what Fujii (2015) describes: "anticipating students' solutions when writing the lesson proposal and evaluating the task during the post-lesson discussion in light of the actual students' responses in the research lesson." (Fujii, 2015, p. 285).

In 5 experimental classrooms where the lesson was carried out, students made a key discovery: "hidden zeros", that shed light onto opacity of the lattice multiplication. This element appeared in similar forms is episodes from all 5 classrooms. From preliminary analyses of the classrooms data, we can state that analyzing different multiplication algorithms by comparing them can indeed bring out awareness of mathematical meanings in children. We would like to discuss whether and how further investigations could clarify way in which this approach can be extended to other algorithms and how it can be used within PD courses exploiting LS.

Relation to the themes chosen

This study contributes to research on the teaching and learning of number flexibility in the context of natural number multiplication through the mastery and conceptual understanding of two algorithms.

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